E PROBLEM SOLVING

When practical problems are converted into algebraic form, a quadratic equation may result. To solve these problems, follow these steps:

Step 1: Carefully **read the question** until you understand the problem.
A **sketch** may be useful.
Step 2: Decide on the **unknown** quantity and label it x, say.
Step 3: Use the information given to write an **equation**.
Step 4: **Solve** the equation.
Step 5: **Check** that any solutions satisfy the equation and are reasonable.
Step 6: Write your answer to the question in **sentence form**.

Example 13 **Self Tutor**

The sum of a number and its square is 42. Find the number.

Let the number be x. Therefore its square is x^2.

$$\begin{aligned} x + x^2 &= 42 \\ \therefore\ x^2 + x - 42 &= 0 \quad \{\text{rearranging}\} \\ \therefore\ (x+7)(x-6) &= 0 \quad \{\text{factorising}\} \\ \therefore\ x = -7 \ \text{ or } \ x &= 6 \end{aligned}$$

So, the number is -7 or 6.

Check: The sum of -7 and its square is $-7 + (-7)^2 = -7 + 49 = 42$ ✓

The sum of 6 and its square is $6 + 6^2 = 6 + 36 = 42$ ✓

1. The sum of a number and its square is 110. Find the number.
2. When 24 is subtracted from the square of a number, the result is five times the original number. Find the number.
3. The sum of two numbers is 6, and the sum of their squares is 28. Find these numbers exactly.
4. Two numbers differ by 7, and the sum of their squares is 29. Find the numbers.

Example 14

Self Tutor

A rectangle has length 5 cm greater than its width, and its area is 84 cm^2. Find the dimensions of the rectangle.

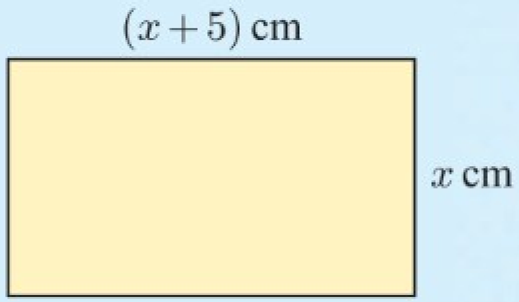
Example 14 **Self Tutor**

A rectangle has length 5 cm greater than its width, and its area is 84 cm^2. Find the dimensions of the rectangle.

Let the width of the rectangle be x cm.

$\therefore$ the length of the rectangle is $(x+5)$ cm.

$$\begin{aligned} \text{Now} \quad \text{area} &= 84\ \text{cm}^2 \\ \therefore \quad x(x+5) &= 84 \\ \therefore \quad x^2+5x &= 84 \\ \therefore \quad x^2+5x-84 &= 0 \\ \therefore \quad (x+12)(x-7) &= 0 \\ \therefore \quad x &= -12 \quad \text{or} \quad 7 \end{aligned}$$



But $x > 0$ as lengths must be positive, so $x = 7$.

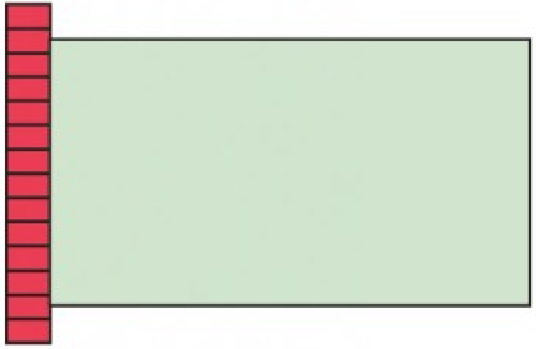
$\therefore$ the rectangle is 7 cm by 12 cm.

5 A rectangle has length 4 cm greater than its width. Find its width given that its area is 96 cm^2.

6 The base of a triangle is 4 m longer than its altitude. The area of the triangle is 70 m^2. Find the triangle's altitude.

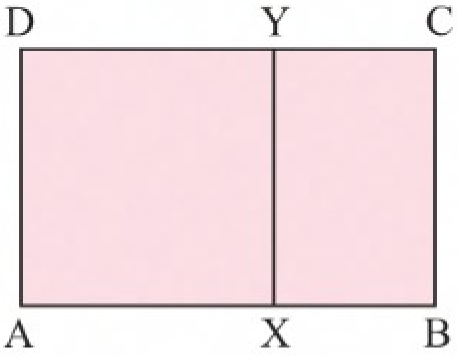
7 A rectangular enclosure is made from 60 m of fencing. The area enclosed is 216 m^2. Find the dimensions of the enclosure.

8 A rectangular garden bed was built against an existing brick wall. 24 m of edging was used to enclose 60 m^2. Find the dimensions of the garden bed to the nearest centimetre.



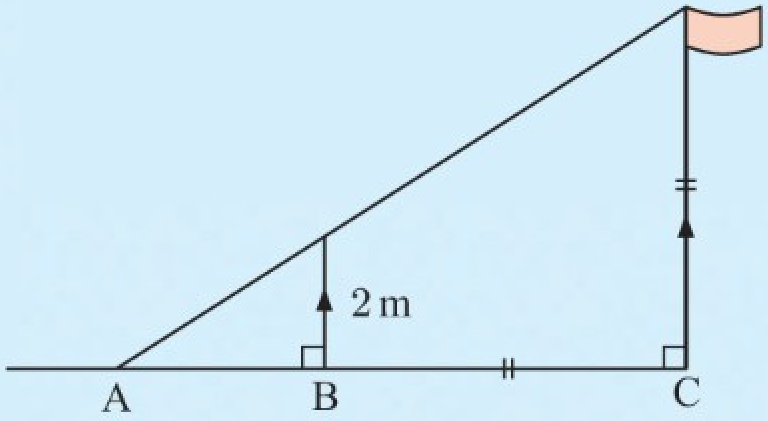
9 A right angled triangle has sides 3 cm and 8 cm respectively less than its hypotenuse. Find the length of the hypotenuse to the nearest millimetre.

10 ABCD is a rectangle in which AB = 21 cm.
The square AXYD is removed and the remaining rectangle has area 80 cm^2.
Find the length of [BC].



Example 15

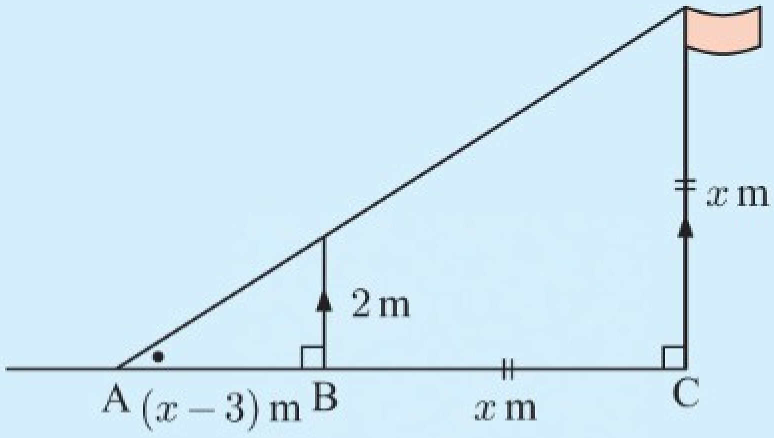
Self Tutor



Given that [AB] is 3 m shorter than [BC], find the height of the flagpole.

Let the height of the flagpole be x m.

$\therefore \quad BC = x$ m and $AB = (x-3)$ m



The triangles are equiangular, so they are similar.

$$\therefore \quad \frac{x}{2} = \frac{(x-3)+x}{x-3}$$

$$\therefore \quad x(x-3) = 2(2x-3)$$

$$\therefore \quad x^2 - 3x = 4x - 6$$

$$\therefore \quad x^2 - 7x + 6 = 0$$

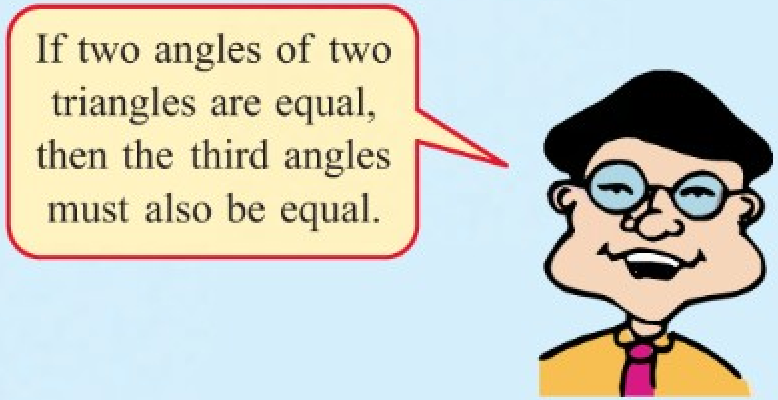
$$\therefore \quad (x-1)(x-6) = 0$$

$$\therefore \quad x = 1 \text{ or } 6$$

But $x - 3 > 0$ as lengths must be positive

$\therefore \quad x = 6$

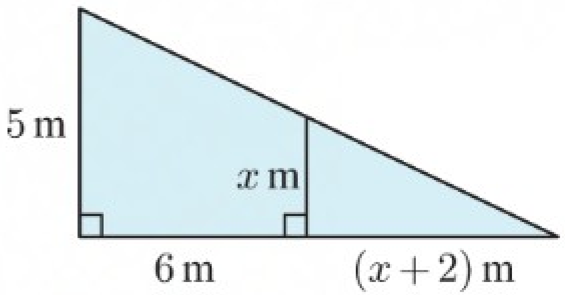
So, the flagpole is 6 m high.



If two angles of two triangles are equal, then the third angles must also be equal.

11 Find x in:

a



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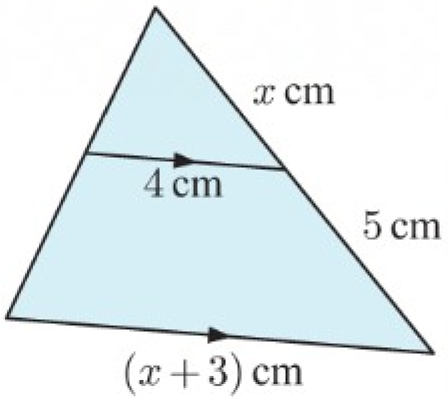
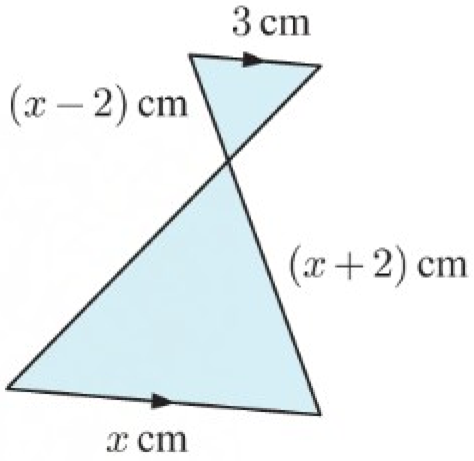


c



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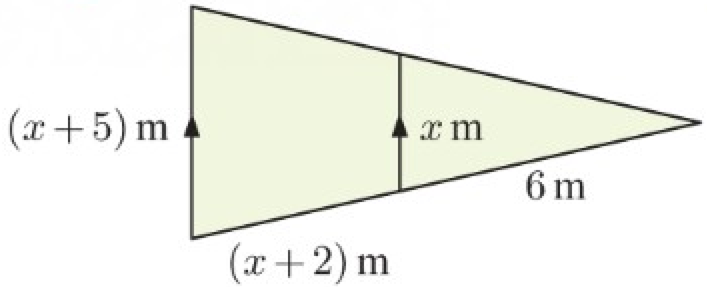

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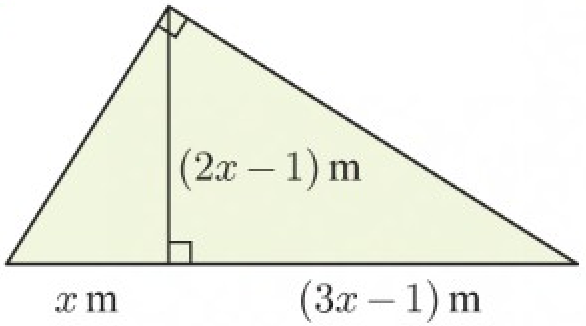

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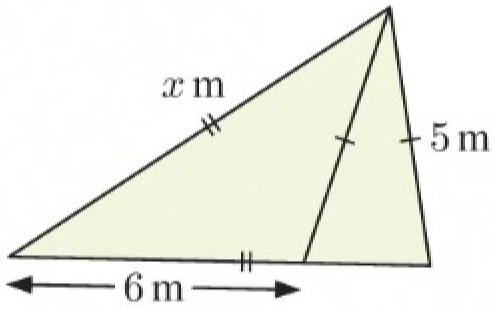

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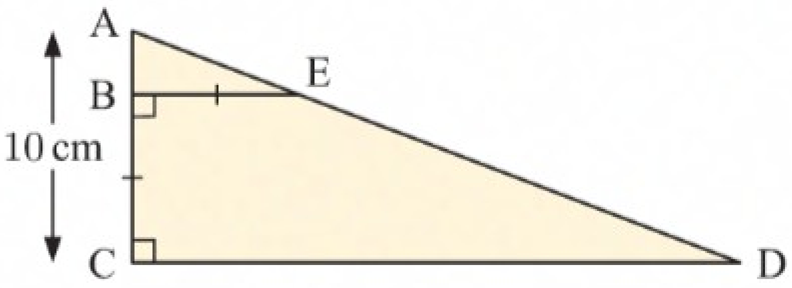
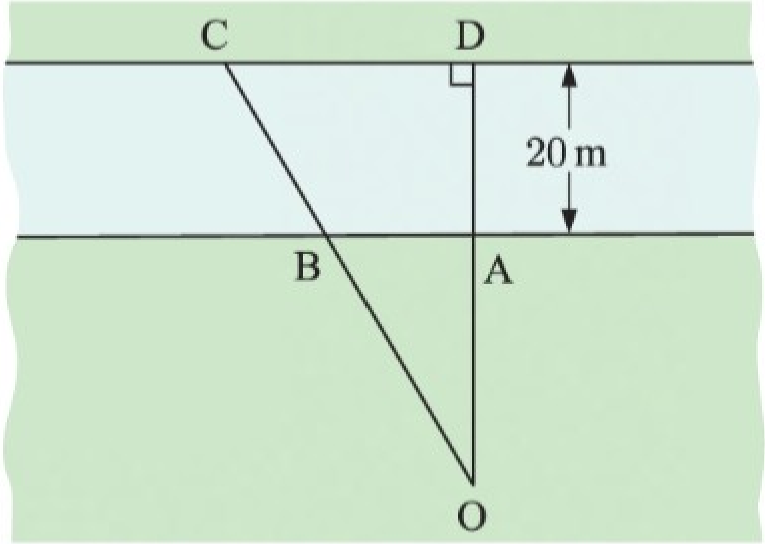


In the figure alongside, [BC] is the same length as [BE], and [CD] is 3 cm more than twice the length of [BE].

Find the length of [BE].

13 A, B, C, and D are posts on the banks of a 20 m wide canal. A and B are 2 m apart, and $\mathrm{OA} = \mathrm{CD}$.

Find the exact distance between C and D.



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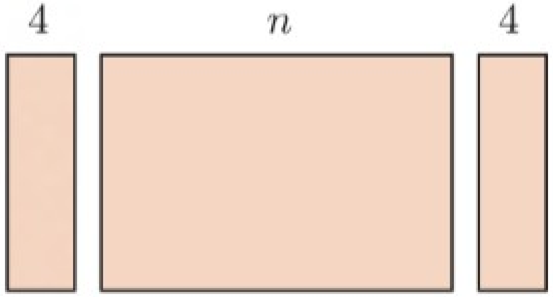


A theatre contains a central block of seats with n seats per row. Blocks on either side contain 4 seats per row. The number of rows is 5 less than the total number of seats per row. In total there are 126 seats in the theatre. Find the value of n.

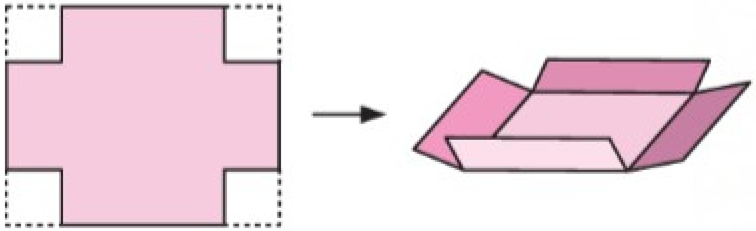
15 The numerator of a fraction is 3 less than the denominator. If the numerator is increased by 6 and the denominator is increased by 5, the fraction is doubled in value. Find the original fraction.

16 At a fruit market, John bought some oranges for a total of \$20. When Jenny visited a different stall, she bought 10 more oranges than John for the same total amount. Given that the difference in price per orange was 10 cents, how many oranges did John purchase?

17 The sum of a number and its reciprocal is $2\frac{1}{12}$. Find the number.

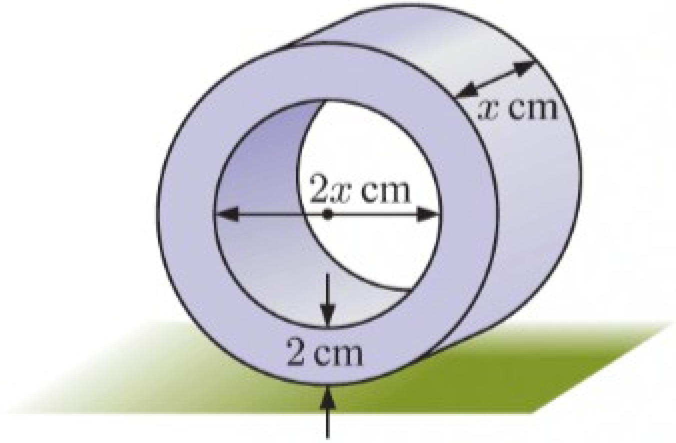
18 Two numbers have a sum of 4, and the sum of their reciprocals is 8. Find the numbers.

19 A sheet of cardboard is 15 cm long and 10 cm wide. It is to be made into an open box with base area 66 cm^2, by cutting out equal squares from the four corners and then bending the edges upwards. Find the size of the squares to be cut out.



20 Answer the **Opening Problem** on page **224**.

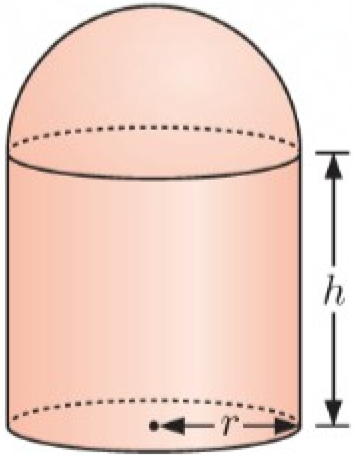
21 A circular magnet has an inner radius of x cm, an outer radius 2 cm larger, and its depth is the same as the inner radius. The total volume of the magnet is 120π cm^3. Find x.



22 A rectangular swimming pool is 12 m long by 6 m wide. It is surrounded by a pavement of uniform width. The area of the pavement is $\frac{7}{8}$ of the area of the pool.

- **a** If the pavement is x m wide, show that the area of the pavement is $(4x^2 + 36x)$ m^2.
- **b** Hence, show that $4x^2 + 36x - 63 = 0$.
- **c** How wide is the pavement?

23



A takeaway milkshake container is cylindrical, with a hemispherical lid on top.

The height h of the container is 7 cm greater than its base radius r.

The surface area of the container and lid is 96π cm^2.

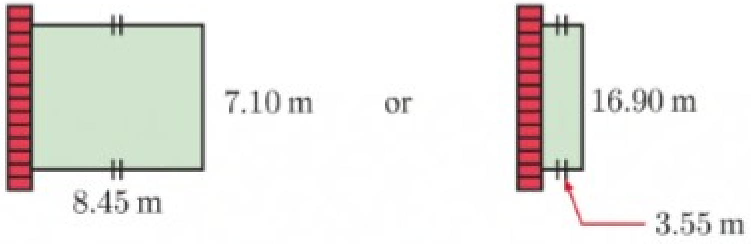
Find the base radius of the container.

1 -11 or 10 **2** -3 or 8 **3** $x = 3 + \sqrt{5}$ and $3 - \sqrt{5}$

4 The numbers are -2 and 5, or 2 and -5.

5 8 cm **6** 10 m **7** 18 m by 12 m

8



9 17.9 cm **10** BC = 16 cm or 5 cm

11 **a** $x = 2$ **b** $x = 5$ **c** $x = 6$ **d** $x = \sqrt{31} - 1$

e $x = \dfrac{3 + \sqrt{5}}{2}$ as $x > \frac{1}{2}$ **f** $x = 3 + \sqrt{34}$

12 BE = 6 cm **13** CD = $(1 + \sqrt{41})$ m **14** $n = 6$

15 $\frac{2}{5}$ or $\frac{-9}{-6}$ **16** 40 oranges **17** $\frac{4}{3}$ or $\frac{3}{4}$

18 The numbers are $\dfrac{4 + \sqrt{14}}{2}$ and $\dfrac{4 - \sqrt{14}}{2}$.

19 Cut out squares with sides 2 cm.

20 **a** $x + 2r = 10 \quad \therefore \quad r = 5 - \dfrac{x}{2}$

b **Hint:** Lawn area = 4 × total of flower bed areas

$$\therefore \quad 2(\pi \times 5^2) = 4 \times \pi r^2$$

$$\therefore \quad 50\pi = 4\pi\left(5 - \frac{x}{2}\right)^2 \quad \text{etc.}$$

c 2.93 m

21 $x = 5$

22 **a** **Hint:** Draw a diagram and show that
area of pavement $= 2\left[x(12 + 2x) + 6x\right]$.

b $4x^2 + 36x = \frac{7}{8}(6 \times 12) = 63$ **c** 1.5 m

23 3.2 cm